N$\emptyset$thing is Logical

Maria Aloni
ILLC & Philosophy
University of Amsterdam
M.D.Aloni@uva.nl

Slides: https://www.marialoni.org/resources/LiRA25.pdf

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N∅thing is logical (Nihil)

- Goal of the project: a formal account of a class of natural language inferences which deviate from classical logic
- Common assumption: these deviations are not logical mistakes, but consequence of pragmatic enrichments (Grice)
- Strategy: develop *logics of conversation* which model next to literal meanings also pragmatic factors and the additional inferences which arise from their interaction
- Novel hypothesis: **neglect-zero** tendency (a cognitive bias rather than a conversational principle) as crucial factor
- Main conclusion: deviations from classical logic consequence of pragmatic enrichments albeit not of the canonical Gricean kind

Nihil team

MA, Anttila, Knudstorp, Degano, Klochowicz & Ramotowska

Collaborators: Bezhanishvili, Bott, Roelofsen, Romoli, Sbardolini, Schlotterbeck, Yan, Yang, Zhou, Wang, . . .

Non-classical inferences

Free choice (FC)

(1) $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

(2) Deontic FC inference [Kamp 1973]

a. You may go to the beach *or* to the cinema.
b. $\rightsquigarrow$ You may go to the beach *and* you may go to the cinema.

(3) Epistemic FC inference [Zimmermann 2000]

a. Mr. X might be in Victoria *or* in Brixton.
b. $\rightsquigarrow$ Mr. X might be in Victoria *and* he might be in Brixton.

Ignorance

(4) The prize is in the attic *or* in the garden $\rightsquigarrow$ speaker doesn't know where
(5) ? I have two *or* three children. [Grice 1989]

- In the standard approach, ignorance inferences are conversational implicatures
- Less consensus on FC inferences analysed as conversational implicatures; grammatical scalar implicatures; semantic entailments; ...

Novel hypothesis: neglect-zero

- FC and ignorance inferences are [$\neq$ semantic entailments]
  - Not the result of Gricean reasoning [$\neq$ conversational implicatures]
  - Not the effect of applications of covert grammatical operators [$\neq$ scalar implicatures]
- But rather a consequence of something else speakers do in conversation, namely,

  NEGLECT-ZERO
  when interpreting a sentence speakers create structures representing reality[1] and in doing so they systematically neglect structures which verify the sentence by virtue of an empty configuration (*zero-models*)

- Tendency to neglect zero-models follows from the difficulty of the cognitive operation of evaluating truths with respect to empty witness sets
- Cognitive difficulty of zero and zero-models confirmed by experimental findings and argued to explain
  1. the special status of 0 among the natural numbers [Nieder, 2016]
  2. why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (*less* vs *more*) [Bott et al., 2019]

[1] Johnson-Laird (1983) *Mental Models*. Cambridge University Press.

Novel hypothesis: neglect-zero

Illustrations (based on predictions of $\text{qBSML}^{\rightarrow}$)[2]

(6) Every square is black. $[\forall x(Sx \rightarrow Bx)]$

a. Verifier: [■, ■, ■]
b. Falsifier: [■, □, ■]
c. Zero-models: [△, △, △]; [▲, ▲, ▲]; . . . $\rightsquigarrow$ there are squares

(7) Less than three squares are black. $[\forall xyz((Sx \wedge Bx \wedge \dots) \rightarrow (x = y \vee \dots))]$

a. Verifier: [■, □, ■]
b. Falsifier: [■, ■, ■]
c. Zero-models: [□, □, □]; [△, △, △]; . . . $\rightsquigarrow$ there are black squares

(8) No squares are black. $[\forall x(Sx \rightarrow \neg Bx)]$

a. Verifier: [□, □, □]
b. Falsifier: [■, □, □]
c. Zero-models: [△, △, △]; [▲, ▲, ▲]; . . . $\rightsquigarrow$ there are squares

(9) Every square is red or white. $[\forall x(Sx \rightarrow (Rx \vee Wx))]$

a. Verifier: [🟥, □, 🟥]
b. Falsifier: [🟥, □, ■]
c. Zero-models: [🟥, 🟥, 🟥]; [□, □, □]; . . . $\rightsquigarrow$ there are white and red squares

- Tendency to neglect zero-models also explains FC & ignorance [MA, *S&P* (2022)]
- Recent priming experiment: (7) & (9) involve the same cognitive process

[2] MA & vOrmondt, Modified numerals and split disjunction. *J of Log Lang and Inf* (2023)

BSML: teams and bilateralism

- Team semantics: formulas interpreted wrt a set of points of evaluation (a team) rather than single ones [Hodges 1997; Väänänen 2007]

Classical vs team-based modal logic

$[M = \langle W, R, V\rangle]$

- Classical modal logic: (truth in worlds)

$$M, w \models \phi, \text{ where } w \in W$$

- Team-based modal logic:

$$M, t \models \phi, \text{ where } t \subseteq W$$

Bilateral state-based modal logic (BSML)

- Teams $\mapsto$ information states [Dekker93; Groenendijk^{+}96; Ciardelli^{+}18]
- Assertion & rejection conditions modelled rather than truth

$$M, s \models \phi, \text{ ``}\phi\text{ is assertable in } s\text{''}, \quad \text{with } s \subseteq W$$
$$M, s \dashv \phi, \text{ ``}\phi\text{ is rejectable in } s\text{''}, \quad \text{with } s \subseteq W$$

- Bilateral negation as polarity switcher [Anderson & Belnap75; Rumfitt00; Fine17]
- Split/tensor disjunction rather than boolean/inquisitive disjunction
- Modalities as in early version of modal inquisitive logic rather than in modal dependence logic
- Neglect-zero tendency modelled by non-emptiness atom (NE) [Yang & Väänänen17]

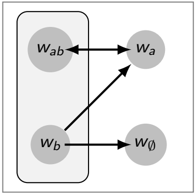
BSML: Classical Modal Logic + NE

Language

$$\phi := p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \text{NE}$$

Bilateral team semantics

Given a Kripke model $M = \langle W, R, V \rangle$ & states $s, t, t' \subseteq W$



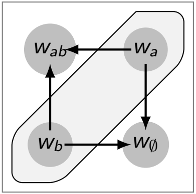
$M, s \models p$	iff	for all $w \in s : V(w, p) = 1$
$M, s \mathrel{=\mkern-4mu|} p$	iff	for all $w \in s : V(w, p) = 0$
$M, s \models \neg\phi$	iff	$M, s \mathrel{=\mkern-4mu|} \phi$
$M, s \mathrel{=\mkern-4mu|} \neg\phi$	iff	$M, s \models \phi$
$M, s \models \phi \vee \psi$	iff	there are $t, t' : t \cup t' = s$ & $M, t \models \phi$ & $M, t' \models \psi$
$M, s \mathrel{=\mkern-4mu|} \phi \vee \psi$	iff	$M, s \mathrel{=\mkern-4mu|} \phi$ & $M, s \mathrel{=\mkern-4mu|} \psi$
$M, s \models \phi \wedge \psi$	iff	$M, s \models \phi$ & $M, s \models \psi$
$M, s \mathrel{=\mkern-4mu|} \phi \wedge \psi$	iff	there are $t, t' : t \cup t' = s$ & $M, t \mathrel{=\mkern-4mu|} \phi$ & $M, t' \mathrel{=\mkern-4mu|} \psi$
$M, s \models \Diamond\phi$	iff	for all $w \in s : \exists t \subseteq R[w] : t \neq \emptyset$ & $M, t \models \phi$
$M, s \mathrel{=\mkern-4mu|} \Diamond\phi$	iff	for all $w \in s : M, R[w] \mathrel{=\mkern-4mu|} \phi$
$M, s \models \text{NE}$	iff	$s \neq \emptyset$
$M, s \mathrel{=\mkern-4mu|} \text{NE}$	iff	$s = \emptyset$ [where $R[w] = \{v \in W \mid wRv\}$]

Entailment: $\phi_1, \ldots, \phi_n \models \psi$ iff for all M, s: $M, s \models \phi_1, \ldots, M, s \models \phi_n \Rightarrow M, s \models \psi$

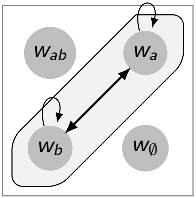
Proof Theory: See Anttila, MA, Yang, *Notre Dame J For Log* (2024).

Team-sensitive constraints on accessibility relation

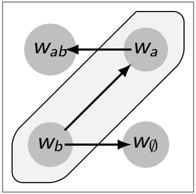
- R is indisputable in (M, s) iff $\forall w, v \in s : R[w] = R[v]$
  $\mapsto$ all worlds in s access exactly the same set of worlds
- R is state-based in (M, s) iff $\forall w \in s : R[w] = s$
  $\mapsto$ all and only worlds in s are accessible within s



(a) indisputable



(b) state-base (& indisputable)



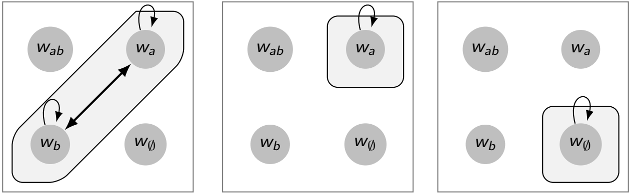
(c) neither

Deontic vs epistemic modals

- Difference deontic vs epistemic modals captured by different properties of accessibility relation:
  - Epistemics: R is state-based
  - Deontics: R is possibly indisputable (e.g. in performative uses)

Neglect-zero effects in BSML: split disjunction

- A state s supports a **disjunction** $(\alpha \vee \beta)$ iff s is the union of two substates, each supporting one of the disjuncts



(d) Verifier (e) **Zero-model** (f) Falsifier

Figure: Models for $(a \vee b)$.

- $\{w_a\}$ verifies $(a \vee b)$ by virtue of an empty witness for the second disjunct, $\{w_a\} = \{w_a\} \cup \emptyset$ & $M, \emptyset \models b$ [$\mapsto$ **zero-model**]
- Main idea: define neglect-zero enrichments, $[\ ]^+$, whose core effect is to rule out such zero-models
- Implementation: $[\ ]^+$ defined using NE ($s \models \text{NE}$ iff $s \neq \emptyset$), which models neglect-zero in the logic

BSML: neglect-zero enrichment

Non-emptiness

NE is supported in a state if and only if the state is not empty

$$\begin{array}{lll} M, s \models \text{NE} & \text{iff} & s \neq \emptyset \\ M, s \mathrel{=\!\!|} \text{NE} & \text{iff} & s = \emptyset \end{array}$$

Neglect-zero enrichment

For NE-free α, $[\alpha]^+$ defined as follows:

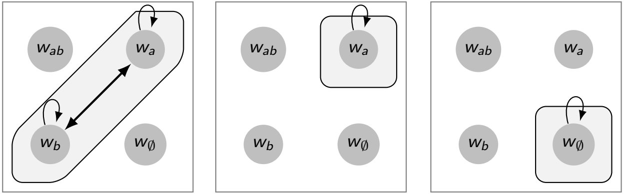
$$\begin{array}{rcl} [p]^+ & = & p \wedge \text{NE} \\ [\neg\alpha]^+ & = & \neg[\alpha]^+ \wedge \text{NE} \\ [\alpha \vee \beta]^+ & = & ([\alpha]^+ \vee [\beta]^+) \wedge \text{NE} \\ [\alpha \wedge \beta]^+ & = & ([\alpha]^+ \wedge [\beta]^+) \wedge \text{NE} \\ [\Diamond\alpha]^+ & = & \Diamond[\alpha]^+ \wedge \text{NE} \end{array}$$

$[\ ]^+$ enriches formulas with the requirement to satisfy NE distributed along each of their subformulas

Neglect-zero effects in BSML: enriched disjunction

- *s* supports an **enriched disjunction** $[\alpha \vee \beta]^+$ iff *s* is the union of two non-empty substates, each supporting one of the disjuncts

$$[\alpha \vee \beta]^+ = (\alpha \wedge \mathrm{NE}) \vee (\beta \wedge \mathrm{NE}) \wedge \mathrm{NE}$$



(a) $\models [a \vee b]^+$ (b) $\not\models [a \vee b]^+$ (c) $=\!\mid [a \vee b]^+$

- An enriched disjunction requires both disjuncts to be live possibilities

(10) It is raining or snowing $\rightsquigarrow$ It might be raining and it might be snowing
$[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta$ (where *R* is state-based)

- Main result: in BSML $[\ ]^+$-enrichment has non-trivial effect only when applied to *positive* disjunctions[3]
  - $\mapsto$ we derive FC and related effects (for enriched formulas);
  - $\mapsto$ $[\ ]^+$-enrichment vacuous under single negation.

[3] MA (2022) Logic and Conversation: the case of free choice. *Semantics and Pragmatics* 15(5).

Neglect-zero effects in BSML: FC predictions

After enrichment

- We derive both wide and narrow scope FC inferences:
  - Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Universal FC: $[\forall x \Diamond(\alpha \vee \beta)]^+ \models \forall x(\Diamond\alpha \wedge \Diamond\beta)$
  - Double negation FC: $[\neg\neg\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide scope FC: $[\Diamond\alpha \vee \Diamond\beta]^+ \models \Diamond\alpha \wedge \Diamond\beta$ (if R is indisputable)
- while no undesirable side effects obtain with other configurations:
  - Dual prohibition: $[\neg\Diamond(\alpha \vee \beta)]^+ \models \neg\Diamond\alpha \wedge \neg\Diamond\beta$

Before enrichment

- The NE-free fragment of BSML is equivalent to classical modal logic (ML):

$$\alpha \models_{BSML} \beta \text{ iff } \alpha \models_{ML} \beta \qquad [\text{if } \alpha, \beta \text{ are NE-free}]$$

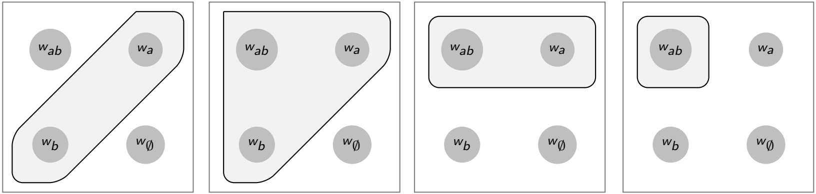
[if α is NE-free: $M, s \models \alpha$ iff for all $w \in s : M, \{w\} \models \alpha$]

- But we can capture the infelicity of epistemic contradictions [Yalcin, 2007] by putting team-based constraints on the accessibility relation:
  1. Epistemic contradiction: $\Diamond\alpha \wedge \neg\alpha \models \bot$ (if R is state-based)
  2. Non-factivity: $\Diamond\alpha \not\models \alpha$

Zero and no-zero models

(M, s) is a zero-model for α iff $M, s \models \alpha$, but $M, s \not\models [\alpha]^+$
(M, s) is a no-zero verifier for α iff $M, s \models [\alpha]^+$

Many no-zero verifiers for enriched disjunction



(d) no-zero & scalar $\models \neg(a \wedge b)$

(e) no-zero, non-scalar $\not\models \neg(a \wedge b)$

(f) no-zero, non-scalar & no-uncertain $\not\models \neg\Box_e a$

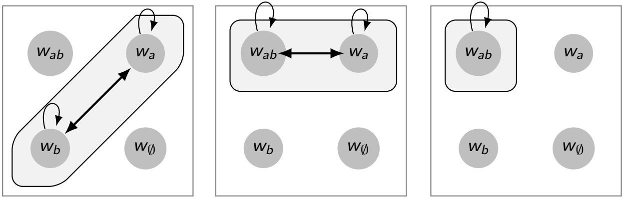
(g) no-zero, non-scalar, no-uncertain & no-split $\models (a \wedge b)$

Figure: Models for enriched $[a \vee b]^+$.

1. Neglect-zero enrichment does not derive scalar implicatures;
2. Neglect-zero enrichment neither derives no-uncertain inferences $\mapsto$ in contrast to standard neo-Gricean approach to ignorance
3. No-split verifiers compatible with neglect-zero enrichments
   - **No-split conjecture**: only no-split verifiers accessible to ‘conjunctive’ pre-school children. [Klochowicz, Sbardolini, MA]

Neglect-zero effects in BSML: possibility vs uncertainty

- More no-zero verifiers for $a \vee b$:



(a) scalar (b) no-uncertain (c) no-split

- Two components of full ignorance ('speaker doesn't know which'):[4]

(11) It is raining or it is snowing $(\alpha \vee \beta) \rightsquigarrow$

a. Uncertainty: $\neg\Box_e\alpha \wedge \neg\Box_e\beta$

b. Possibility: $\Diamond_e\alpha \wedge \Diamond_e\beta$ (equiv $\neg\Box_e\neg\alpha \wedge \neg\Box_e\neg\beta$)

- **Fact:** Only possibility derived as neglect-zero effect:
  - $[a \vee b]^+ \models \Diamond_e a \wedge \Diamond_e b$ (if R is state-based)
  - $\{w_{ab}, w_a\} \models [a \vee b]^+$, but $\not\models \neg\Box_e a$
  - $\{w_{ab}\} \models [a \vee b]^+$, but $\not\models \neg\Box_e a$; $\not\models \neg\Box_e b$

[4] Degano, Marty, Ramotowska, MA, Breheny, Romoli, Sudo. *Nat Lang Sem*, 2025.

Two derivations of full ignorance

1. Standard neo-Gricean derivation [Sauerland 2004]

(i) Uncertainty derived through quantity reasoning

(12)	$\alpha \vee \beta$	ASSERTION
(13)	$\neg\Box_e\alpha \wedge \neg\Box_e\beta$	UNCERTAINTY (from QUANTITY)

(ii) Possibility derived from uncertainty and quality about assertion

(14)	$\Box_e(\alpha \vee \beta)$	QUALITY ABOUT ASSERTION
(15)	$\Rightarrow \Diamond_e\alpha \wedge \Diamond_e\beta$	POSSIBILITY

2. Neglect-zero derivation

(i) Possibility derived as neglect-zero effect

(16)	$\alpha \vee \beta$	ASSERTION
(17)	$\Diamond_e\alpha \wedge \Diamond_e\beta$	POSSIBILITY (from NEGLECT-ZERO)

(ii) Uncertainty derived from possibility and scalar reasoning

(18)	$\neg(\alpha \wedge \beta)$	SCALAR IMPLICATURE
(19)	$\Rightarrow \neg\Box_e\alpha \wedge \neg\Box_e\beta$	UNCERTAINTY

Neo-Gricean vs neglect-zero explanation

Contrasting predictions of competing accounts of ignorance

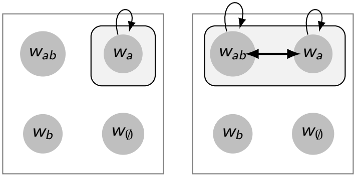
- Neo-Gricean: No possibility without uncertainty
- Neglect-zero: Possibility derived independently from uncertainty

Experimental findings [Degano *et al* 2025]

- Using adapted mystery box paradigm, compared conditions in which
  - both uncertainty and possibility are false [**zero-model**]
  - uncertainty false but possibility true [no-zero, no-uncertain model]
- Less acceptance when possibility is false (95% vs **44%**)

⇒ Evidence that possibility can arise without uncertainty

- A challenge for the traditional neo-Gricean approach

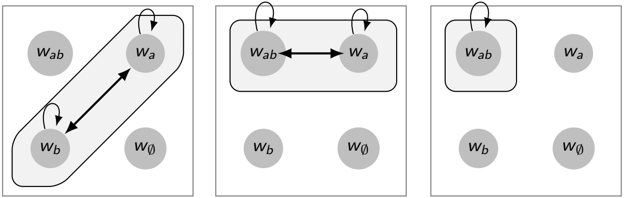


(d) **zero-model** (e) no-uncertain

Figure: Models for $(a \vee b)$

Neglect-zero and no-split

- More no-zero verifiers for $a \vee b$:



(a) scalar (b) no-uncertain (c) no-split

- No-split verifiers: singletons, no contrasting possibilities entertained
- Split verifiers: multi-membered sets, multiple alternatives processed in a parallel fashion $\mapsto$ a cognitively taxing operation

NO-SPLIT CONJECTURE [Klochowicz, Sbardolini & MA]
the ability to split states (parallel processing of contrasting alternatives) is acquired late

- Implementation: uses flattening operator F

$$M, s \models \mathrm{F}\phi \text{ iff for all } w \in s : M, \{w\} \models \phi$$

- The combination of neglect-zero & no-split explains non-classical inferences observed in pre-school children

No-split and the acquisition of 'or'

- Basic data: pre-school children interpret *or* as *and* [e.g., Singh *et al* 2016, Cochard 2023, Bleotu *et al* 2024]:

  (20) The boy is holding an apple or a banana = The boy is holding an apple and a banana $\alpha \vee \beta = \alpha \wedge \beta$

  (21) Liz can buy a croissant or a donut = Liz can buy a croissant and a donut $\Diamond(\alpha \vee \beta) = \Diamond(\alpha \wedge \beta)$

  (22) The boy is not holding an apple or a banana = The boy is neither holding an apple nor a banana $\neg(\alpha \vee \beta) = \neg\alpha \wedge \neg\beta$

- Combination of no-split and **no-zero** yields conjunctive *or*:

$$
\begin{aligned}
[\mathrm{F}(\alpha \vee \beta)]^+ &\equiv \alpha \wedge \beta \\
[\Diamond \mathrm{F}(\alpha \vee \beta)]^+ &\equiv \Diamond\alpha \wedge \Diamond\beta \\
[\neg \mathrm{F}(\alpha \vee \beta)]^+ &\equiv \neg\alpha \wedge \neg\beta
\end{aligned}
$$

- Nihil explanation: beside neglecting zero-models, children further lack the ability to split states, i.e. have difficulties in processing contrasting possibilities in a parallel fashion

Illustration: combination of no-split and no-zero yields conjunctive *or*

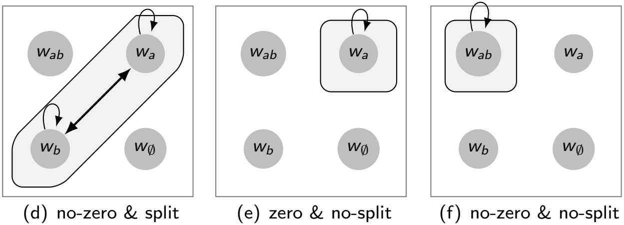


Figure: Models for $(a \vee b)$

Predicted inferences

- No-zero & split: $[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta;\ \not\models \alpha \wedge \beta$ [adult-like]
- Zero & (no-)split: $(\mathrm{F})(\alpha \vee \beta) \not\models \Diamond_e \alpha \wedge \Diamond_e \beta;\ \not\models \alpha \wedge \beta$ [logician]
- No-zero & no-split: $[\mathrm{F}(\alpha \vee \beta)]^+ \models \alpha \wedge \beta$ [‘conjunctive’ children]

BSML & related systems: information states vs possible worlds

- Failure of bivalence in BSML

$$M, s \not\models p \;\&\; M, s \not\models \neg p, \text{ for some info state } s$$

- Info states: less determinate than possible worlds
    - just like truthmakers, situations, possibilities, ...
- Technically:
    - Truthmakers/possibilities: points in a partially ordered set
    - Info states: sets of possible worlds, also elements of a partially ordered set, the Boolean lattice $Pow(W)$
- Thus systems using these structures are closely connected, although might diverge in motivation:
    - Truthmaker & possibility semantics: description of ontological structures in the world
    - BSML & inquisitive semantics: explaining patterns in inferential & communicative human activities
- NEXT:
    - Comparison via translations in Modal Information Logic [vBenthem19]

BSML & related systems: comparisons via translation

- Modal Information Logic (MIL) (van Benthem, 1989, 2019):[5] common ground where related systems can be interpreted and their connections and differences can be explored
- Goal: translations into MIL of the following systems:
    - BSML
    - Truthmaker semantics (Fine)
    - Possibility semantics (Humberstone, Holliday)
    - Inquisitive semantics (Ciardelli, Groenendijk & Roelofsen)

  (cf. Gödel's (1933) translation of intuitionistic logic into modal logic)
- Here focus on propositional fragments
    - disjunction
    - negation
- (Based on work in progress with Søren B. Knudstorp, Nick Bezhanishvili, Johan van Benthem and Alexandru Baltag)

[5] Johan van Benthem (2019) Implicit and Explicit Stances in Logic, *Journal of Philosophical Logic*.

Modal Information Logic (MIL)

Language

$$\phi \quad ::= \quad p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \langle sup \rangle \phi\psi$$

where $p \in A$.

Models and interpretation

Formulas are interpreted on triples $M = (X, \leq, V)$ where $\leq$ is a partial order

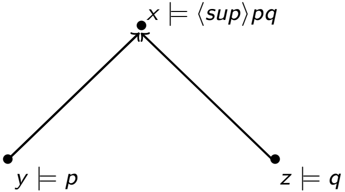
$$\begin{array}{rll}
\mathcal{M}, x \models p & \text{iff} & x \in V(p) \\
\mathcal{M}, x \models \neg\phi & \text{iff} & \mathcal{M}, x \not\models \phi \\
\mathcal{M}, x \models \phi \wedge \psi & \text{iff} & \mathcal{M}, x \models \phi \ \text{ and } \ \mathcal{M}, x \models \psi \\
\mathcal{M}, x \models \phi \vee \psi & \text{iff} & \mathcal{M}, x \models \phi \ \text{ or } \ \mathcal{M}, x \models \psi \\
\mathcal{M}, x \models \langle sup \rangle \phi\psi & \text{iff} & \text{there are } y, z : x = sup_{\leq}(y, z) \ \& \ \mathcal{M}, y \models \phi \ \& \ \mathcal{M}, z \models \psi
\end{array}$$

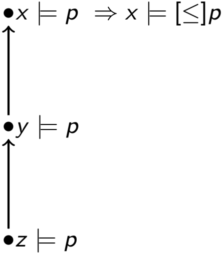
$[\leq]\phi = \neg\langle sup \rangle(\neg\varphi)\top$

$$\mathcal{M}, x \models [\leq]\phi \quad \text{iff} \quad \text{for all } y : y \leq x \ \Rightarrow \ \mathcal{M}, y \models \phi$$

Modal Information Logic (MIL)

Examples





Translations into Modal Information Logic

- Possibility semantics (Humberstone, Holliday)[6]

$$\begin{array}{rcl} & \vdots & \\ tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\ tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\ tr(\phi \vee \psi) & = & [\leq]\langle\leq\rangle(tr(\phi) \vee tr(\psi)) \\ & \vdots & \end{array}$$

- Inquisitive semantics (Groenendijk, Roelofsen and Ciardelli)

$$\begin{array}{rcl} & \vdots & \\ tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\ tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\ tr(\phi \vee \psi) & = & tr(\phi) \vee tr(\psi) \\ & \vdots & \end{array}$$

[6] Johan van Benthem, Nick Bezhanishvili, Wesley H. Holliday, A bimodal perspective on possibility semantics, *Journal of Logic and Computation*, Volume 27, Issue 5, July 2017, Pages 1353–1389.

Translations into Modal Information Logic

- Truthmaker semantics (Fine): $\leq$ is "part of" relation[7]

$$
\begin{aligned}
&\dots\\
(\neg\phi)^+ &= (\phi)^-\\
(\neg\phi)^- &= (\phi)^+\\
(\phi \vee \psi)^+ &= (\phi)^+ \vee (\psi)^+\\
(\phi \vee \psi)^- &= \langle sup\rangle(\phi)^-(\psi)^-\\
(\phi \wedge \psi)^+ &= \langle sup\rangle(\phi)^+(\psi)^+\\
(\phi \wedge \psi)^- &= (\phi)^- \vee (\psi)^-
\end{aligned}
$$

- BSML: $\leq$ is subset relation $\subseteq$, ...

$$
\begin{aligned}
&\dots\\
(\neg\phi)^+ &= (\phi)^-\\
(\neg\phi)^- &= (\phi)^+\\
(\phi \vee \psi)^+ &= \langle sup\rangle(\phi)^+(\psi)^+\\
(\phi \vee \psi)^- &= (\phi)^- \wedge (\psi)^-\\
(\phi \wedge \psi)^+ &= (\phi)^+ \wedge (\psi)^+\\
(\phi \wedge \psi)^- &= \langle sup\rangle(\phi)^-(\psi)^-\\
&\dots
\end{aligned}
$$

Goal: with $\mathbf{0}$ (classical modal logic);[8] without $\mathbf{0}$ (BSML*).

[7] van Benthem, Implicit and Explicit Stances in Logic, *Journal of Philosophical Logic* (2019).

[8] Humberstone, Operational Semantics for Positive R. *Notre Dame J of Form Log* (1988).

Disjunction and Negation

- Three notions of disjunction expressible in MIL:
  - Boolean disjunction: $\phi \vee \psi$
    [classical logic, intuitionistic logic, inquisitive logic]
  - Lifted/tensor/split disjunction: $\langle sup \rangle \phi\psi$
    [BSML, dependence logic, team semantics, operational semantics for Positive R]
  - Cofinal disjunction: $[co](\phi \vee \psi)$ (where $[co]\phi =: [\leq]\langle\leq\rangle\phi$)
    [possibility semantics, dynamic semantics]
- Three notions of negation:
  - Boolean negation: $\neg\phi$
    [classical logic, ...]
  - Bilateral negation: $(\neg\phi)^+ = (\phi)^-$ & $(\neg\phi)^- = (\phi)^+$
    [truthmaker semantics, BSML, ...]
  - Intuitionistic-like negation: $[\leq]\neg\phi$
    [possibility semantics, inquisitive semantics, intuitionistic logic]
- Some combinations:
  - Boolean disjunction + boolean negation $\mapsto$ classical logic
  - Boolean notions in other combinations can generate non-classicality:
    - Boolean disjunction + intuitionistic negation $\mapsto$ intuitionistic logic
  - Classicality also generated by non-boolean combinations:
    - Split disjunction + bilateral negation (classical fragm. BSML)

Conclusions

- FC and ignorance: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Nihil proposal: non-classical inferences consequences of cognitive biases
  - FC and ignorance as neglect-zero effects

    Literal meanings (NE-free fragment) + cognitive factors (NE) $\Rightarrow$ FC & possibility inferences

  - Conjunctive *or* as no-zero + no-split effect

    Literal meanings (NE-free fragment) + cognitive factors (NE, F) $\Rightarrow$ conjunctive *or*

- Implementation in BSML^{F} (a team-based modal logic)
- Differences but also interesting connections with related systems
- MIL useful framework for comparisons via translations

Collaborators & related (future) research

Logic

Proof theory (Anttila, Yang); expressive completeness (Anttila, Knudstorp); bimodal perspective (Knudstorp, Baltag, van Benthem, Bezhanishvili); qBSML (van Ormondt); BiUS & qBiUS (MA); typed BSML (Muskens); connexive logic (Knudstorp & MA);. . .

Language

FC cancellations (Pinton, Hui); modified numerals (vOrmondt); attitude verbs (Yan); conditionals (Flachs); questions (Klochowicz); quantifiers (Klochowicz, Bott, Schlotterbeck); indefinites (Degano); homogeneity (Sbardolini); acquisition (Klochowicz, Sbardolini); experiments (Degano, Klochowicz, Ramotowska, Bott, Schlotterbeck, Marty, Breheny, Romoli, Sudo); . . .

THANK YOU![9]

[9] This work was supported by NWO OC project *Nothing is Logical* (grant no 406.21.CTW.023).

Appendix

Novel hypothesis: neglect-zero

Comparison with competing accounts of FC inference

	NS FC	Dual Prohib	Universal FC	Double Neg	WS FC
Neo-Gricean	yes	yes	no	?	no
Grammatical	yes	yes*	yes	no*	no*
Semantic	yes	no*	yes	no*	no
Neglect-zero	yes	yes	yes	yes	yes

Argument in favor of neglect-zero hypothesis

- Empirical coverage: FC sentences give rise to a complex pattern of inferences

(23) a. $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Narrow Scope FC]
b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$ [Dual Prohibition]
c. $\forall x \Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$ [Universal FC]
d. $\neg\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Double Negation FC]
e. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Wide Scope FC]

- Captured by neglect-zero approach implemented in BSML[10]
- Most other approaches need additional assumptions

[10] MA (2022). Logic and conversation: the case of FC. *Sem & Pra*, 15(5).

The data

(24) Dual Prohibition [Alonso-Ovalle 2006, Marty *et al.* 2021]

a. You are not allowed to eat the cake or the ice-cream.
$\rightsquigarrow$ You are not allowed to eat either one.

b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$

(25) Universal FC [Chemla 2009]

a. All of the boys may go to the beach or to the cinema.
$\rightsquigarrow$ All of the boys may go to the beach and all of the boys may go to the cinema.

b. $\forall x\Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$

(26) Double Negation FC [Gotzner *et al.* 2020]

a. Exactly one girl cannot take Spanish or Calculus.
$\rightsquigarrow$ One girl can take neither of the two and each of the others can choose between them.

b. $\exists x(\neg\Diamond(\alpha(x) \vee \beta(x)) \wedge \forall y(y \neq x \rightarrow \neg\neg\Diamond(\alpha(y) \vee \beta(y)))) \rightsquigarrow$
$\exists x(\neg\Diamond\alpha(x) \wedge \neg\Diamond\beta(x) \wedge \forall y(y \neq x \rightarrow (\Diamond\alpha(y) \wedge \Diamond\beta(y))))$

(27) Wide Scope FC [Zimmermann 2000, Hoeks *et al.* 2017]

a. Detectives may go by bus or they may go by boat.
$\rightsquigarrow$ Detectives may go by bus and may go by boat.

b. Mr. X might be in Victoria or he might be in Brixton.
$\rightsquigarrow$ Mr. X might be in Victoria and might be in Brixton.

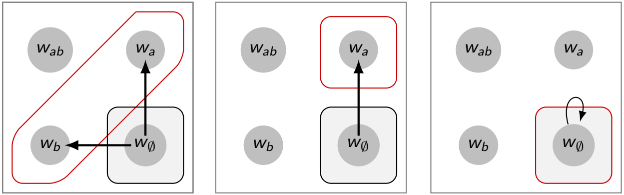
c. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

BSML: illustration free choice facts

- **Free choice** results rely on relational notion of modality:
  - A state s supports $\Diamond\phi$ iff for every w in s there is a non-empty subset of the set of worlds accessible from w which supports ϕ:

$$M, s \models \Diamond\phi \text{ iff } \forall w \in s : \exists t \subseteq R[w] : t \neq \emptyset \ \& \ M, t \models \phi$$

- **Narrow and wide scope** FC (the latter if R is indisputable)



(a) No zero verifier (b) Zero-model (c) Falsifier

Figure: Models for $\Diamond(a \vee b)$ and $\Diamond a \vee \Diamond b$ (R is indisputable).

BSML: illustration free choice facts

- **Free choice** results rely on relational notion of modality:
  - A state s supports $\Diamond\phi$ iff for every w in s there is a non-empty subset of the set of worlds accessible from w which supports ϕ:

$$M, s \models \Diamond\phi \text{ iff } \forall w \in s : \exists t \subseteq R[w] : t \neq \emptyset \;\&\; M, t \models \phi$$

- **Failure of wide scope** FC (R is not indisputable).

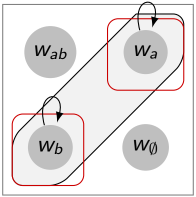
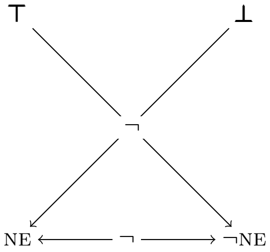


Figure: $s \models [\Diamond a \vee \Diamond b]^+$, but $s \not\models \Diamond a$ (and also $s \not\models [\Diamond(a \vee b)]^+$)

BSML: tautologies and contradictions

- Strong
  - $\top := p \vee \neg p$ — always supported
  - $\bot := \mathrm{NE} \wedge \neg\mathrm{NE}$ — never supported
- Weak
  - NE — supported by all non-empty states
  - $\neg\mathrm{NE}$ — supported only by empty state

Effect of negation



- Failure of replacement under $\neg$:
  - $\neg\top \equiv \neg\mathrm{NE}$, but $\neg\neg\top \not\equiv \neg\neg\mathrm{NE}$;
  - $\neg\bot \equiv \mathrm{NE}$, but $\neg\neg\bot \not\equiv \neg\mathrm{NE}$.